

# The distance duality relation and the temperature profile of Galaxy Clusters

Shuo Cao and Zong-Hong Zhu\*

*Department of Astronomy, Beijing Normal University, Beijing, 100875, China \**

## Abstract

The validity of distance duality relation,  $\eta = D_L(z)(1+z)^{-2}/D_A(z) = 1$ , an exact result required by the Etherington reciprocity theorem, where  $D_A(z)$  and  $D_L(z)$  are the angular and luminosity distances, plays an essential part in cosmological observations and model constraints. In this paper, we investigate some consequences of such a relation by assuming  $\eta$  a constant or a function of the redshift. In order to constrain the parameters concerning  $\eta$ , we consider two groups of cluster gas mass fraction data including 52 X-ray luminous galaxy clusters observed by Chandra in the redshift range  $0.3 \sim 1.273$  and temperature range  $T_{\text{gas}} > 4$  keV, under the assumptions of two different temperature profiles [1]. We find that the constant temperature profile is in relatively good agreement with no violation of the distance duality relation for both parameterizations of  $\eta$ , while the one with temperature gradient (the Vikhlinin et al. temperature profile) seems to be incompatible even at 99% CL.

PACS numbers: 98.80.-k; 95.36.+x

---

\*Electronic address: zhuzh@bnu.edu.cn

## I. INTRODUCTION

The reciprocity relation [2,3],  $D_L/D_A(1+z)^{-2} = 1$ , also known as reciprocity law or reciprocity theorem, which links the luminosity distance  $D_L(z)$  with the angular diameter distance  $D_A(z)$  at redshift  $z$ , is a fundamental result for astronomical observations in cosmology. It is extensively used in various domains ranging from gravitational lensing studies to analyses of the cosmic microwave black body radiation observations, as well as for galaxy and galaxy cluster observations [4-22].

As is known to everyone, one fundamental hypothesis in General Relativity is that light travels along null geodesics in a Riemannian spacetime (see [23] for instance), based on which the reciprocity law is generally established. Actually, the reciprocity relation is also valid for other cosmological models based on Riemannian geometry besides the FLRW cosmology. The only condition is that sources and observers are connected by null geodesics in a general Riemannian spacetime. Up to now, diverse astrophysical mechanisms such as gravitational lensing and dust extinction have been proved to be capable of causing obvious deviation from the distance duality [24] and testing this equality with high accuracy can also provide a powerful probe of exotic physics [25]. Therefore, the verification of the observational validity of the reciprocity law is, perhaps, one of the major challenging open problems in modern cosmology. Due to this, many previous works have been done regarding some attempts to test the identity with diverse model parameterizations of  $\eta$  [25-27]. In this paper, we assume that the reciprocity relation can be a constant or a function of the redshift, namely:

$$\frac{D_L}{D_A}(1+z)^{-2} = \eta_c, \frac{D_L}{D_A}(1+z)^{-2} = \eta(z), \quad (1)$$

where  $\eta_c$  denotes a constant around the standard case ( $\eta_c = 1$ ) and  $\eta(z)$  quantifies a possible time-dependent departing from the standard result ( $\eta = 1$ ).

For instance, Bassett & Kunz (2004) [25] used current supernovae Ia data as a measurement of the luminosity distance  $D_L$ , and estimated  $D_A$  from FRIIb radio galaxies [28] and ultra compact radio sources [9,29,30] to detect possible new physics.

Meanwhile, observations from Sunyaev-Zeldovich effect (SZE) and X-ray surface

brightness from galaxy clusters may also provide us a test for the distance duality relation. In order to quantify the  $\eta$  parameter, Uzan et al. (2004) [26] fixed  $D_A(z)$  by using the cosmic concordance  $\Lambda$ CDM model [31] with  $D_A(z)$  from 18 galaxy clusters [32]. They got  $\eta = 0.91^{+0.04}_{-0.04}$  ( $1\sigma$ ) by assuming  $\eta$  a constant. More recently, De Bernardis et al. (2006) [33] have also carried out semblable works using the angular diameter distances from 38 galaxy clusters provided by the sample of Ref. [34], and they obtained  $\eta = 0.97^{+0.03}_{-0.03}$  at  $1\sigma$  in the context of  $\Lambda$ CDM model. More recently, Holanda et al. (2010) [27] used the reciprocity relation to show how one may assess the cluster geometry by using the constraint provided by the reciprocity relation, with the final result that the elliptical geometry for clusters as advocated by De Filippis et al. is more favorable.

In this paper, instead of testing the reciprocity relation, we take it for granted in order to show how we may assess the cluster temperature profiles by using the constraint provided by the reciprocity relation. A robust method is that we could obtain the observational  $\eta(z)$  at redshift  $z$  from the cluster gas mass fraction,  $f_{\text{gas}} = M_{\text{gas}}/M_{\text{tot}}$ , which could be inferred from X-ray observations of clusters of galaxies. Notice that the presence of temperature profiles plays an essential part in the determination of measured total mass, the fraction of mass in stars and, hence, the gas fraction. In this context, with two groups of cluster gas mass fraction data from the Chandra satellite [1], we test the influence on the distance duality relation when different assumptions about the cluster temperature profiles are considered. Our analysis here will be based on two parametric representations defined by Eq.(1), namely: I.  $\eta = \eta_c$ ; II.  $\eta(z) = 1 + \eta_a z$ . The first expression is on the assumption that  $\eta$  does not evolve with redshifts while the second is a continuous and smooth linear one-parameter expansion.

Obviously, the above parameterizations are inspired by similar expressions for  $w(z)$ -the equation of state in the constant  $w$  and time-varying dark energy models (see Ref.[17,35,36]). More importantly, for our subsequent analysis, concerning a given data set, the likelihood of  $\eta_c$  or  $\eta_a$  must be peaked (or  $\Delta\chi^2$  minimizes equivalently) at  $\eta_c = 1$  or  $\eta_a = 0$  in order to satisfy the Etherington theorem. As we shall see, assuming the strict validity of the reciprocity theorem, our analysis indicates that the constant temperature assumption tends to be more compatible with no violation of the reciprocity relation. In

principle, this kind of result is an interesting example of how a cosmological condition correlates to the local physics.

This paper is organized as follows. In Section II we present two samples of cluster gas mass fraction data from 52 X-ray luminous galaxy clusters obtained by Chandra. We then briefly describe the analysis method and show the results of constraining parameters including  $\eta_c$  and  $\eta_a$  in Section III. Finally, we summarize our main conclusions in Section IV.

## II. GALAXY CLUSTER SAMPLES

In order to constrain the possible values of  $\eta_c$  and  $\eta_a$ , we consider two samples of cluster gas mass fraction data from 52 X-ray luminous galaxy clusters obtained by Chandra in the redshift range  $0.3 \sim 1.273$  and temperature range  $T_{\text{gas}} > 4$  keV, one of which is estimated by assuming a constant temperature equal to  $T_{\text{gas}}$  [1], while the other is on the hypothesis of a different temperature profile which reproduces well the temperature gradients in nearby relaxed systems [37]:

$$T(r) = 1.23 \frac{(x/0.045)^{1.9} + 0.45}{(x/0.045)^{1.9} + 1} \frac{T_{\text{gas}}}{[(x/0.6)^2 + 1]^{0.45}}, \quad (2)$$

where  $x = r/R_{500}$ . Moreover, Ettori et al. (2009) [1] found this temperature profile might increase  $f_{\text{gas}}(< R_{500})$  by about 16 percent with respect to the estimates obtained under the assumption of isothermality. Because of this, in the following analysis, we make use of the gas mass fraction data obtained with different temperature profiles to limit parameters. We recall here that in the overall sample of 52 objects at  $z > 0.3$ ,  $(H_0, \Omega_m = 1 - \Omega_\Lambda) = (70 \text{ km s}^{-1} \text{ Mpc}^{-1}, 0.3)$  is assumed, which is also used in the computations throughout this paper.

In Fig. 1 we plot two samples of cluster gas fraction data from 52 X-ray luminous galaxy clusters obtained in the redshift range  $0.3 \sim 1.273$  [1], which were also used as a proxy for the cosmological parameters such as  $\Omega_m$  and  $w$  [1]. However, since these samples are endowed with different temperature profile assumptions, our major interest here is to confront these underlying hypotheses with the validity of the reciprocity relation.

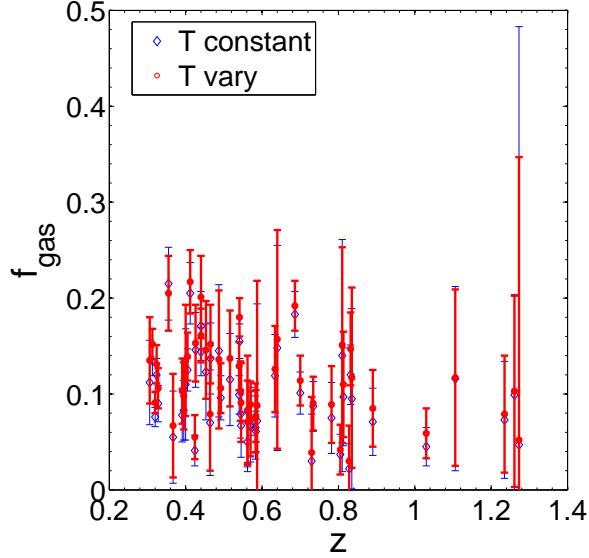


FIG. 1: The gas mass fraction data [1]. The open blue diamonds and red circles with the associated error bars stand, respectively, for the constant and Vikhlinin et al. temperature profile samples.

Thereafter, our attention is paid to the process of extracting  $\eta$  from the gas mass fraction. As is well known, old, relaxed, rich galaxy clusters should provide us with a characteristic sample of the matter content of the universe if they are large enough. Moreover, the baryonic fraction  $f_{gas}$ , should remain the same during cosmic evolution if the baryonic-to-total mass ratio in clusters is equal to the cosmological baryonic mass fraction ratio  $\Omega_b/\Omega_m$ . Since clusters are observed at different redshifts and the reconstructed  $f_{gas}$  depends on the assumed distance to the cluster, this data can be used to constrain  $\eta$ . Following Allen et al. (2008) [38], we compute the gas mass fraction given by

$$f_{gas}(z) = \frac{KA\gamma b}{1+s} \frac{\Omega_b}{\Omega_m} \left( \frac{d_A^{ref}(z)}{d_A(z)} \right)^{1.5}, \quad (3)$$

where five parameters ( $K, A, \gamma, b, s$ ) are related to modeling the cluster gas mass fraction. For example, the factor  $A$  is the angular correction factor and is close to unity for all redshifts [38].

The depletion parameter  $b$  indicates the amount of cosmic baryons thermalized within

Cluster	Parameter	Allowance
Calibration/modelling	K	$1.0 \pm 0.1$
Non-thermal pressure	$\gamma$	$1.0 < \gamma < 1.1$
Gas depletion	b	$0.874 \pm 0.023$
Stellar mass	s	$0.18-0.012T_{gas}$
Standard $\Omega_b h^2$	$\Omega_b h^2$	$0.0233 \pm 0.0008$
$\Omega_m$	$\Omega_m$	0.3
Hubble constant	$H_0$	$70 km s^{-1} Mpc^{-1}$

TABLE I: Summary of the standard systematic allowances and priors included in the Chandra  $f_{gas}$  analysis.

the cluster potential. This “bias” factor was modeled as  $b = b_0(1 + a_b z)$  with the uniform priors  $0.65 < b_0 < 1.0$  and  $-0.1 < a_b < 0.1$  suggested by cosmological simulations [38]. More recently, according to various SPH and Eulerian simulations of a single cluster presented in the Santa Barbara Comparison Project, Frenk et al. (1999) [39] gained  $b = 0.92 \pm 0.07$ . We adopt, in our paper, the result from the simulated dataset in Ref. [1]:  $b_{500} = 0.874 \pm 0.023$ , which is only related to the most massive systems, subjected only to the gravitational heating, high-temperature objects. The parameter  $s = f_{star}/f_{gas}$  denotes the baryon gas mass fraction in stars. Allen et al. (2008) [38] modeled it as  $s = s_0(1 + a_s z)$ , using the uniform prior  $-0.2 < a_s < 0.2$  as well as the Gaussian prior  $s_0 = 0.13 \pm 0.01$ . However, recent works found that an additional intracluster light (ICL) at very low surface brightness might also contribute to the total cold baryonic content of galaxy clusters:  $f_{cold} = f_{star} + f_{ICL}$  [40,41]. More recently, Lagana et al. (2008) [42] discussed in detail the baryonic content of five massive galaxy clusters, including an ICL contribution. They concluded that the stellar-to-gas mass ratio relating to the gas temperature could be expressed through the relation:  $f_{cold} = f_{star} + f_{ICL} = (0.18 - 0.012T_{gas}) f_{gas}$ , where  $T_{gas}$  is measured in keV and  $T_{gas} > 4$  keV is one selection criterion. This parametrization was extensively discussed in Ref. [1] and is also adopted in our work.  $\Omega_b$  is the present dimensionless density parameter of the baryonic matter and the WMAP observations

give  $\Omega_b h^2 = 0.0233 \pm 0.0008$  [31].  $D_A^{\text{ref}}$  is the angular diameter distance computed in a reference, spatially-flat  $\Lambda$ CDM model with  $\Omega_\Lambda = 0.7$  ( $\Omega_m = 0.3$ ), and  $D_A$  is the true angular diameter distance. The complete set of standard priors and allowances included in the gas mass fraction ( $f_{\text{gas}}$ ) analysis are summarized in Table I.

With  $\eta_{\text{obs}}^2 = D_A^{\text{Cluster}}(z)/D_A^{\text{Th}}(z)$  (see Ref. [26]), Eq.(3) could be rewritten as:

$$\eta_{\text{obs}}^3 = \frac{K A \gamma b(z)}{1 + s(z)} \frac{\Omega_b}{\Omega_m} f_{\text{gas}}^{-1}. \quad (4)$$

In our statistical analysis (see the next section), the observational  $\eta_{\text{obs}}^3$  and its corresponding uncertainty are also calculated through Eq.(4).

### III. ANALYSIS AND RESULTS

Let us now estimate the  $\eta_c$  and  $\eta_a$  parameters for both samples in two parameterizations for  $\eta = D_L(z)(1+z)^{-2}/D_A(z)$ , e.g.,  $\eta = \eta_c$ ,  $\eta(z) = 1 + \eta_a z$ . To begin with, we evaluate the likelihood distribution function  $e^{-\chi^2/2}$ , where

$$\chi^2 = \sum \frac{[\eta^3 - \eta_{\text{obs}}^3]^2}{\sigma_{\eta_{\text{obs}}^3}^2}, \quad (5)$$

with  $\eta_{\text{obs}}^3$  gained from the  $f_{\text{gas}}$  data [Eq.(4)]. As for  $\sigma_{\eta_{\text{obs}}^3}$ , the uncertainty of  $\eta_{\text{obs}}^3$ , is calculated through the propagation of uncertainty statics with a combination of the observational  $f_{\text{gas}}$ , the best fit values of parameters such as  $K$ ,  $\gamma$  as well as their corresponding standard systematic allowances (Fig. 1 and Table I).

In Fig. 1 and 3, we plot the likelihood distributions on the  $\eta_c - \Delta\chi^2$  and  $\eta_a - \Delta\chi^2$  planes for both samples. We obtain  $\eta_c = 0.955_{-0.015}^{+0.015}$  ( $\chi_{d.o.f.}^2 = 47.25/52$ ) and  $\eta_a = -0.082_{-0.029}^{+0.029}$  ( $\chi_{d.o.f.}^2 = 49.50/52$ ) at 68.3% CL for the Vikhlinin et al. temperature profile sample and  $\eta_c = 0.981_{-0.015}^{+0.015}$  ( $\chi_{d.o.f.}^2 = 63.44/52$ ) and  $\eta_a = -0.026_{-0.034}^{+0.034}$  ( $\chi_{d.o.f.}^2 = 64.28/52$ ) at 68.3% CL for the constant temperature sample. We can see that the  $\eta$  from the latter is in agreement with no violation of the reciprocity relation while the former, where a Vikhlinin et al. temperature profile is assumed to describe the clusters, is incompatible even at 99% CL. Therefore, we have not found any evidence for distance duality violation when the constant temperature sample is considered. However, the same kind of analysis

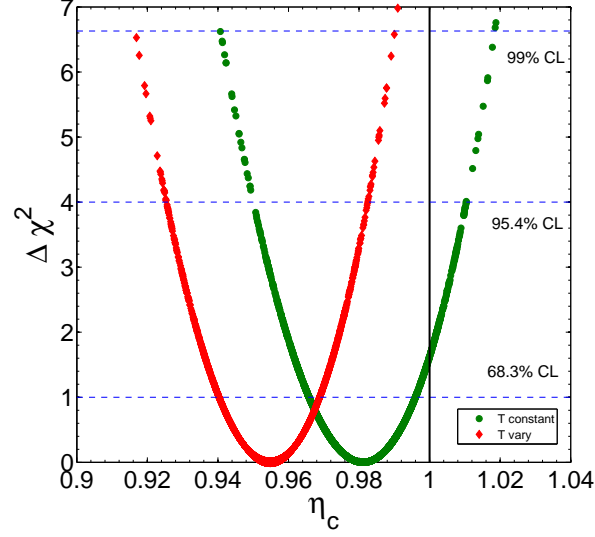


FIG. 2: The  $\eta_c - \Delta\chi^2$  plane for the parameterization  $\eta = \eta_c$ . The filled (green) circles and filled (red) diamonds stand for the constant and Vikhlinin et al. temperature profile samples, respectively.

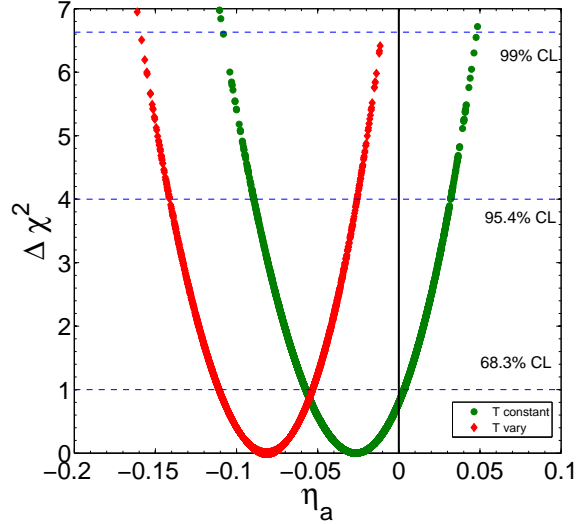


FIG. 3: The  $\eta_a - \Delta\chi^2$  plane for the parameterization  $\eta(z) = 1 + \eta_a z$ . The denotation of different data is the same as Fig.2.



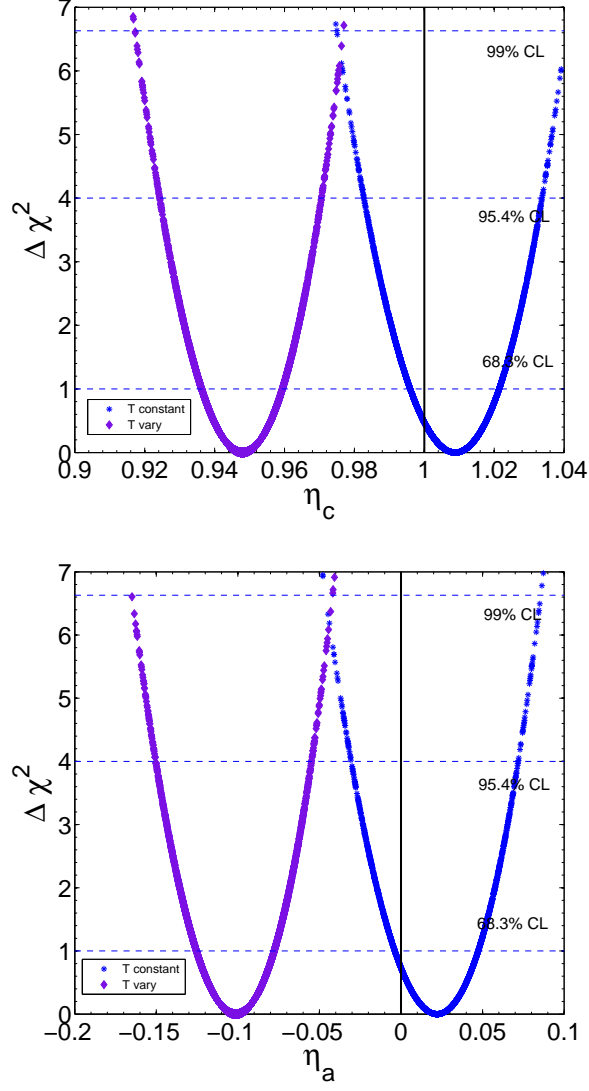


FIG. 4: The  $\eta_c - \Delta\chi^2$  planes for two parameterizations  $\eta = \eta_c$  and  $\eta(z) = 1 + \eta_a z$  with marginalized  $f_{gas}$  parameters. The blue stars and filled purple diamonds stand for the constant and Vikhlinin et al. temperature profile samples, respectively.

shown in Fig. 2 and 3 points to be in contradiction with the Vikhlinin et al. temperature profile hypothesis assumed in the Vikhlinin et al. sample. Furthermore, considering that many parameters listed in Table I are fixed to their best-fit values, we perform a MCMC analysis and marginalize over these nuisance parameters [43]. The constraint results shown in Fig. 4 further confirm our conclusions.

#### IV. CONCLUSIONS

In this paper, we have explored the consequences of the distance duality relation  $\eta = D_L(1+z)^{-2}/D_A$  based on two samples of cluster gas mass fraction data from 52 X-ray luminous galaxy clusters obtained by Chandra in the redshift range  $0.3 \sim 1.273$  and temperature range  $T_{\text{gas}} > 4$  keV [1]. We discussed the consistency between the strict validity of the distance duality relation and the assumptions about temperature profiles (the constant and Vikhlinin et al. temperature profiles) used to describe the galaxy clusters. The  $\eta$  parameter was parametrized in two different functional forms,  $\eta = \eta_c$  and  $\eta = 1 + \eta_a z$ .

By comparing the constant and Vikhlinin et al. temperature profile samples, we show that the constant temperature profile is more consistent with no violation of the distance duality relation in the context of two groups of cluster gas mass fraction data. In the case of constant temperature sample (see Fig. 2 and 3) we find  $\eta_c = 0.981^{+0.015}_{-0.015}$  and  $\eta_a = -0.026^{+0.034}_{-0.034}$  for constant and linear parametrizations, respectively. On the other hand, the Vikhlinin et al. temperature profile model (see Fig. 2 and 3) seems to be marginally incompatible with  $\eta_c = 0.955^{+0.015}_{-0.015}$  and  $\eta_a = -0.082^{+0.029}_{-0.029}$  for constant and linear parameterizations, respectively. Our analysis indicates that the constant temperature assumption tends to be more compatible with the Etherington theorem at 68.3% CL (between 68.3% and 95.4% CL for the first parametrization) compared with the Vikhlinin et al. temperature profile hypothesis, while the latter is incompatible even at 99% CL. The results with marginalized  $f_{\text{gas}}$  parameters (Fig. 4) further confirm these conclusions.

In summary, we find that, according to the statistical analysis presented here, the constant temperature profile of clusters tends to be more consistent with no violation of the distance duality relation regarding two groups of cluster gas mass fraction data. This reinforces the interest in the observational search for such kind of data from clusters at high redshifts. With better data, the method proposed here based on the validity of the distance duality relation should lead to improved limits on the temperature profiles of gas in galaxy clusters.

## Acknowledgments

This work was supported by the National Natural Science Foundation of China under the Distinguished Young Scholar Grant 10825313 and Grant 11073005, and by the Ministry of Science and Technology national basic science Program (Project 973) under Grant No. 2007CB815401.

- 
- [1] Ettori, S., et al. 2009, AAP, 501, 61 [arXiv:0904.2740]
  - [2] Linder, E. V. 1988, A&A, 206, 190
  - [3] Schneider, P., Ehlers, J., & Falco, E. E. 1992, Gravitational Lenses, Springer-Verlag, Berlin
  - [4] Zhu, Z.-H., & Wu, X.-P. 1997, A&A, 324, 483
  - [5] Mao, S., & Schneider, P. 1998, MNRAS, 295, 587
  - [6] Jin, K.-J., Zhang, Y.-Z., & Zhu, Z.-H. 2000, PLA, 264, 335
  - [7] Keeton, C. R. 2001, ApJ, 561, 46
  - [8] Kochanek, C. S., & White, M. 2001, ApJ, 559, 531
  - [9] Lima, J. A. S., & Alcaniz, J. S. 2002, ApJ, 566, 15
  - [10] Chae, K.-H., & Mao, S. 2003, ApJ, 599, L61
  - [11] Lima, J. A. S., Cunha, J. V., & Alcaniz, J. S. 2003, PRD, 68, 023510
  - [12] Ofek, E. O., Rix, H.-W., & Maoz, D. 2003, MNRAS, 343, 639
  - [13] Chae, K.-H. 2005, ApJ, 630, 764
  - [14] Rusin, D., & Kochanek, C. S. 2005, ApJ, 623, 666
  - [15] Koopmans, L. V. E., Treu, T., Bolton, A. S., Burles, S., & Moustakas, L. A. 2006, ApJ, 649, 599
  - [16] Treu, T., Koopmans, L. V. E., Bolton, A. S., Burles, S., & Moustakas, L.A. 2006, ApJ, 640, 662
  - [17] Cunha, J. V., Marassi, L., & Santos, R.C. 2007, IJMPD, 16, 403
  - [18] Fu, L. et al., 2008, A&A, 479, 9
  - [19] Zhu, Z.-H., & Mauro Sereno, 2008, A&A, 831, 835

- [20] Zhu, Z.-H., et al. 2008, A&A, 483, 15
- [21] Mantz, A., Allen, S. W., Rapetti, D., & Ebeling, H. 2010, MNRAS, 406, 1759  
[arXiv:0909.3098]
- [22] Komatsu, E. et al., 2010, arXiv:1001.4538 (WMAP collaboration)
- [23] Ellis, G. F. R. 2007, GRG, 39, 1047
- [24] Corasaniti, P. S. 2006, MNRAS, 372, 191
- [25] Bassett, B. A., & Kunz, M. 2004, PRD, 69, 101305
- [26] Uzan, J. P., Aghanim, N., & Mellier, Y. 2004, PRD, 70, 083533
- [27] Holanda, R. F. L., Lima, J. A. S., & Ribeiro, M. B. arXiv:1003.5906v1
- [28] Daly, R. A., & Djorgovski, S.G. 2003, ApJ, 597, 9
- [29] Lima, J. A. S., & Alcaniz, J. S. 2000, A&A., 357, 393
- [30] Santos, R. C., & Lima, J. A. S. 2008, PRD, 77, 083505
- [31] Spergel, D. N., et al. 2003, APJS, 148, 175; Spergel, D. N., et al. 2007, ApJS, 170, 377
- [32] Reese, E. D., et al. 2002, ApJ, 581, 53
- [33] De Bernardis, F., Giusarma, E., & Melchiorri, A. 2006, IJMPD, 15, 759
- [34] Bonamente M., et al. 2006, ApJ, 647, 25
- [35] Linder, E. V. 2003, PRL, 90, 091301
- [36] Padmanabhan, T., & Choudury, R. 2003, MNRAS, 344, 823
- [37] Vikhlinin, A., Kravtsov, A., Forman, W., Jones, C., Markevitch, M., Murray, S.S., & Van  
Speybroeck, L. 2006, ApJ, 640, 691.
- [38] Allen, S. W., et al. 2008, MNRAS, 383, 879
- [39] Frenk, C.S., et al. 1999, ApJ, 525, 554
- [40] Murante, G., et al. 2004, ApJ, 607, L83
- [41] Murante, G., Giovalli, M., Gerhard, O., Arnaboldi, M., Borgani, S., & Dolag, K. 2007,  
MNRAS, 377, 2
- [42] Lagana, T.F., Lima Neto, G.B., Andrade-Santos, F., & Cypriano, E.S. 2008, A&A, 485,  
633
- [43] Ganga, K., Ratra, B., Gunderson, J. O., & Sugiyama, N. 1997, ApJ, 484, 7